# FMEA and PFD Evaluation of Ball Valve

Doc. No.: JDV20.2-2019

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# Table of content

# 1. Purpose and scope

General two options exist when doing safety integrity level assessment for hardware product like valves.

Option 1: Hardware assessment according to IEC 61508

Option 1 is a hardware assessment according to the relevant functional safety standard like IEC 61508. The hardware assessment consists of a FMEA to determine the fault behavior and the failure rates of the devices, which are then used to calculate the Safe Failure Fraction (SFF), and the average Probability of Failure on Demand ($PFD_{AVG}$).

Option 2: Hardware assessment with proven-in-use consideration according to IEC 61508 / IEC 61511.

Option 2 is an assessment according to relevant functional safety standard like IEC 61508. The hardware assessment consists of a FMEA to determine the fault behavior and the failure rates of the devices, which are then used to calculate the Safe Failure Fraction (SFF), and the average Probability of Failure on Demand ($PFD_{AVG}$). In addition this option consists of an assessment of the proven-in-use documentation of the device.

This assessment shall be done according to option 2.

This document shall include the FMEA of Valve, and assess whether the device meet the average Probability of Failure on Demand ($PFD_{AVG}$) requirements and the architectural constraints for SIL 2 sub-systems according to IEC 61508 / IEC 61511.

# 2. Referenced documents

## 2.1. Codes & standards

- 0 **IEC 61508 edition 2.0** – Functional safety of electrical/electric/programmable electronic safety-related systems;
- 0 **IEC 61511 first edition 2003-03 –** Functional safety – Safety instrumented systems for the process industry sector;
- 0 **API Q1 9th edition**– Specification for quality management system requirements for manufacturing organizations for the petroleum and natural gas industry.

## 2.2. Documents used as proven-in-use evidences

- 0 After-sales service records
- 0 Delivery records of 2011, 2012 and 2013
- 0 Statistics of field-feed-back tracking; sold and return devices

# 3. Product Description

# 4. Failure modes, effects, and analysis

## 4.1. Description of the failure categories

Three Safety Instrumented Functions (SIF) are defined in Safety Requirement Specification for Valve:

a)  Valve to open on demand;
b)  Valve to close (full stroke) on demand;
c)  Valve to close (tight shutoff).

SIF a is considered to have same failure categories with SIF b. The failure categories for SIF c shall be separately defined.

| Failure categories | SIF a&b | SIF c |
|---|---|---|
| Fail-safe state | Valve to open or close | Valve to close (tight shutoff) |
| Fail dangerous | Valve does not respond to a demand from the process to open or close | Valve does not respond to a demand from the process to tightly shutoff. |
| Fail no effect | Failure of a component that is part of the safety function but that has no effect on the safety functions | Failure of a component that is part of the safety function but that has no effect on the safety functions |
| Not considered | Not considered means that this failure mode was not considered. When calculating the SFF this failure mode is divided into 50% safe failure and 50% dangerous failures. | Not considered means that this failure mode was not considered. When calculating the SFF this failure mode is divided into 50% safe failure and 50% dangerous failures. |
| Not part | Failures of a component which is not part of the safety function but part of the product and is listed for completeness. When calculating the SFF this failure mode is not taken into account. It is also not part of the total failure rate. | Failures of a component which is not part of the safety function but part of the product and is listed for completeness. When calculating the SFF this failure mode is not taken into account. It is also not part of the total failure rate. |

Table 1. Failure categories of FMEAs for valve

## 4.2. Methodology – FMEA, failure rates

### 4.2.1.    FMEA

A Failure modes and effects analysis (FMEA) is a systematic way to identify and evaluate the effects of different component failure modes, to determine what could eliminate or reduce the change of failure, and to document the system in consideration.

### 4.2.2.    Failure rates

The failure rate used in this FEMA are collected from the statistics of field experiences. The user of these numbers is responsible for determining their applicability to any particular environment. Some industrial plan sites have high levels of stress. Under those

conditions the failure rate data is adjusted to a higher value to account for the specific conditions of the plant.

### 4.2.3. Assumptions

The following assumptions have been made during the FMEA:

0    Failure rates are constant, wear out mechanisms are not included.
0    Propagation of failures is not relevant.
0    The repair time after a safe failure is 8 hours.
0    All modules are operated in the low demand mode of operation.
0    External power supply failure rates are not included.
0    5% of the valves delivered in 2011 is assumed to have not been put in use, and 10% for 2012, 15% for 2013.
0    Due to the failures are usually reported during commissioning or beginning stage, the valves with complaint are considered delivered in the same year of complaint reported.
0    Due to lacking of accurate records regarding to when the valves are put in use and when the failures happen, the successive working hours are calculated by using average method, ie. a valve delivered in 2013 is considered has 4380 successive working hours (0.5 year), and 13140 hours (1.5 year)for a valve delivered in 2012, 21900 hours for 2011.

### 4.2.4. Abbreviation

DOP          Delayed operation, including fail to respond and any other circumstances of failing to open or close
ELP          External leakage
PST          Partial stroke test
LCP          Valve leakage in closed position

### 4.3. Summary of sales and after-sales data

| Year | 2014 | | |
|---|---|---|---|
| Delivered Number | 817 | | |
| Complained Number | 6 | | |
| Failure category | Dangerous | Safe | No effect |
| SIF a & b | 0 | 4 | 2 |
| SIF c | 2 | 2 | 2 |
| Year | 2015 | | |
| Delivered Number | 877 | | |
| Complained Number | 34 | | |
| Failure category | Dangerous | Safe | No effect |
| SIF a & b | 9 | 2 | 23 |
| SIF c | 9 | 2 | 23 |
| Year | 2016 | | |
| Delivered Number | 227 | | |
| Complained Number | 67 | | |
| Failure category | Dangerous | Safe | No effect |
| SIF a & b | 0 | 2 | 65 |
| SIF c | 2 | 0 | 65 |

Table 2. Data summary

**NOTE**

Due to lacking of effective method to monitor the working performance of the product supplied for foreign projects, all the data above stated is collected from domestic sales and after-sales records.

### 4.4. FMEA Table

Refer to Appendix A.

## 5. Result of assessment

### 5.1. Methodology – Markov process

According to the assumptions stated in 4.2.3, the total successive working hours T=5.1168E07 hours. And the $\lambda$ and SFF are as listed in following table.

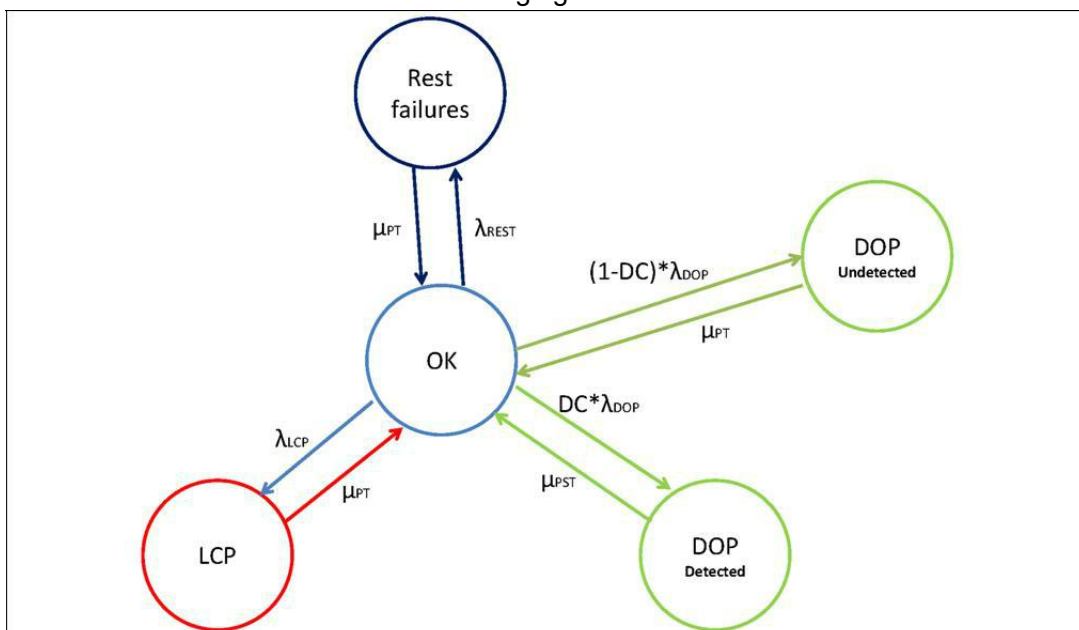| | $\lambda_{safe}$ (per $10^9$ hour) | $\lambda_{dangerous}$ (per $10^9$ hour) |
|---|---|---|
| SIF a&b | 293 | 329 |
| SIF c | 146 | 475 |

Table 3. $\lambda_{safe}$ and $\lambda_{dangerous}$ for each SIF

The PFD$_{AVG}$ calculations for three SIFs are done in one Markov model, the $\lambda$ data are re-summarized as following table.

| $\lambda_{DOP}$ | $\lambda_{LCP}$ | $\lambda_{REST}$ |
|---|---|---|
| 1.76E-07 per hour | 7.82E-08 per hour | 1.84E-06 per hour |

Table 4. Reorganization of $\lambda$ values

The Markov model is showed in following figure.



**Abbreviations:**

$\lambda_{DOP}$ Failure rate of delayed operation (including not responding etc.)
$\lambda_{LCP}$ Failure rate of leakage in closed position
$_{PT}$     Repair rate of proof test

| μPST | Repair rate of partial stroke test |
|------|------------------------------------|
| DC   | Diagnostic Coverage (45.2%)        |

**Figure 3: Markov model for valve**

**NOTE**

1) For SIF a&b, λdangerous is equivalent to λDOP, while λsafe equals to the sum of λLCP and λREST. For SIF c, λdangerous is equivalent to the sum of λDOP and λLCP, λsafe equals to λREST.

2) The Diagnostic Coverage data used in this document is quoted from Volume 3 of *Safety Equipment Reliability Handbook*, 3rd edition, exida. The diagnostic only applies to those failure related to the full stroke, failure related to the tight shutoff will not be detected by the partial stroking of the valve.

3) The failures are considered independent from each other, the states with two or more failures occurring at same time period are not taken in into account.

4) The proof test is assumed to be perfect. And except the failures detected during partial stroke test (PST), all other failures will be detected and restored during proof test.

The PDF$_{AVG}$ (average unavailability ) was calculated based on equations as follow.

$$PFD_{AVG}(T) = \frac{1}{T} \sum_{k=1}^{n} q_k MCT_k(T)$$

Where $q_k = 1$ if the system is unavailable in state k, and $q_k = 0$ otherwise; MCT is the Mean Cumulated times spent in the states. Refer to IEC 61508-6 for details.

## 5.2. PFD$_{AVG}$ Calculations for SIF a & b

### 5.2.1. PFD$_{AVG}$ calculations (without PST function)

The PFD$_{AVG}$ was calculated for three different proof test intervals using the Markov model without taking the PST into account. The PFD$_{AVG}$ values for three proof test intervals (1 year, 2 years and 5 years) are displayed in following table.

|              | TI=1 year, MTTR=24 hr | TI=2 year, MTTR=24 hr | TI=5 year, MTTR=24 hr |
|--------------|-----------------------|-----------------------|-----------------------|
| **PFD$_{AVG}$** | 0.007825537602        | 0.007818795991        | 0.008813905087        |

### 5.2.2. PFD$_{AVG}$ calculations (with PST function)

The PFD$_{AVG}$ with PST taken into account was calculated by using the multi-phase Markov model. The partial stroke tests are singular points along the time, and the system will start again from a new beginning state when the PST is performed. The new beginning state will be calculated from the previous state before PST by using a linking matrix [L].

The PFD$_{AVG}$ values for three proof test intervals - TI (1 year, 2 years and 5 years) with two different PST intervals - PSTI (3 months and 6 months) are displayed in following table. (DC=45.2%, refer to 5.1).

|  | TI=1 year, MTTR=24 hr | TI=2 year, MTTR=24 hr |
|---|---|---|
| **PSTI=3 months** | PFD$_{AVG}$ = 0.0008615831273 | PFD$_{AVG}$ = 0.001614029029 |
| **PSTI=6 months** | PFD$_{AVG}$ = 0.0009515547176 | PFD$_{AVG}$ = 0.001702524390 |

## 5.3. PFD$_{AVG}$ Calculations for SIF c

### 5.3.1. PFD$_{AVG}$ calculations (without PST function)

Different from SIF a & b, the occurrence of either LCP or DOP will be regarded as dangerous failure of SIF c. The calculation was done based on the same Markov model stated in Figure 1.

|  | TI=1 year, MTTR=24 hr | TI=2 year, MTTR=24 hr | TI=5 year, MTTR=24 hr |
|---|---|---|---|
| **PFD$_{AVG}$** | 0.008059109870 | 0.008071594209 | 0.009842307347 |

### 5.3.2. PFD$_{AVG}$ calculations (with PST function)

The PFD$_{AVG}$ with PST taken into account was calculated by using the multi-phase Markov model. The partial stroke tests are singular points along the time, and the system will start again from a new beginning state when the PST is performed. The new beginning state will be calculated from the previous state before PST by using a linking matrix [L].

The PFD$_{AVG}$ values for three proof test intervals - TI (1 year, 2 years and 5 years) with two different PST intervals - PSTI (3 months and 6 months) are displayed in following table. (DC=45.2%, refer to 5.1).

|  | TI=1 year, MTTR=24 hr | TI=2 year, MTTR=24 hr |
|---|---|---|
| **PSTI=3 months** | PFD$_{AVG}$ = 0.001486710499 | PFD$_{AVG}$ = 0.002850611022 |
| **PSTI=6 months** | PFD$_{AVG}$ = 0.001576668885 | PFD$_{AVG}$ = 0.002939083193 |

## 6. Conclusion

The calculated PFD$_{AVG}$ values are within the allowed range for SIL 2 according to Table 2 of IEC 61508-1 which is as follow.

| Safety integrity level (SIL) | Average probability of a dangerous failure on demand of the safety function (PFD$_{AVG}$) |
|---|---|
| 4 | $\geq 10^{-5}$ to $< 10^{-4}$ |
| 3 | $\geq 10^{-4}$ to $< 10^{-3}$ |
| 2 | $\geq 10^{-3}$ to $< 10^{-2}$ |
| 1 | $\geq 10^{-2}$ to $< 10^{-1}$ |

**Table 5. Table 2 of IEC 61508-1**

**Appendix A:** FMEA of valve

| N° | Component | Function | Failure mode | Cause | Effect | Detection Mode | Failure category for SIF a&b | Failure category for SIF c |
|---|---|---|---|---|---|---|---|---|
| **FMEA of Valve** | | | | | | | | |
| 1 | Body | Contain the process pressure | Fracture | Material Defection | ELP | Undetectable during PST | No effect | No effect |
| | | | Distortion | Material Defection, Overpressure | DOP | Detectable during PST | Dangerous | Dangerous |
| | | | Explosion | Material Defection, Overpressure, Corrosion, Erosion | No function can be implemented any more | Undetectable during PST | Dangerous | Dangerous |
| 2 | Ball | Shut off the medium | Fracture | Material Defection, Overpressure | LCP | Undetectable during PST | No effect | Dangerous |
| | | | Distortion 1 | Material Defection, Overpressure | DOP | Detectable during PST | Dangerous | Dangerous |
| | | | Distortion 2 | Material Defection, Overpressure | LCP | Undetectable during PST | No effect | Dangerous |
| | | | Erosion 1 | Pollution | LCP | Undetectable during PST | No effect | Dangerous |
| | | | Erosion 2 | Pollution | DOP | Detectable during PST | Dangerous | Dangerous |
| 3 | Sealing ring | Provide sealing between seat and body | Break | Material Defection | LCP | Undetectable during PST | No effect | Dangerous |
| 4 | Seat | Shut off the medium | Fracture | Material Defection, Overpressure | LCP | Undetectable during PST | No effect | Dangerous |
| | | | Distortion 1 | Material Defection, Overpressure | DOP | Detectable during PST | Dangerous | Dangerous |
| | | | Distortion 2 | Material Defection, Overpressure | LCP | Undetectable during PST | No effect | Dangerous |
| | | | Erosion 1 | Pollution | LCP | Undetectable during PST | No effect | Dangerous |
| | | | Erosion 2 | Pollution | DOP | Detectable during PST | Dangerous | Dangerous |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | Spring | Provide sealing force between ball and seat | Distortion or Break | Material Defection | LCP | Undetectable during PST | No effect | Dangerous |
| 6 | Thrust washer | Decrease the friction | Break | Material Defection | DOP | Detectable during PST | Dangerous | Dangerous |
| 7 | Bearing | Decrease the friction | Break | Material Defection | DOP | Detectable during PST | Dangerous | Dangerous |
| 8 | Grease fitting | Grease injection for emergency | Choked | Pollution | LCP under fire | Undetectable during PST | No effect | Dangerous |
| 10 | Stem | Drive the ball to required position | Break | Material Defection | DOP | Detectable during PST | Dangerous | Dangerous |
| 11 | Screw | Fasten the stem sealing devices | Break | Material Defection | ELP | Undetectable during PST | No part | No part |
| 12 | Mounting plate | Mounting the actuator | Distortion | Excessive actuator output | DOP | Detectable during PST | Dangerous | Dangerous |
| 13 | Packing | Seal the stem | Wear off | Material Defection, Over-duty use | ELP | Undetectable during PST | No effect | No effect |
| | | | Distortion | Incorrect installation | DOP | Detectable during PST | Dangerous | Dangerous |
| 14 | Bolt/Nut | Fasten the body and bonnet | Fracture | Material Defection | ELP | Undetectable during PST | No effect | No effect |
| | | | Simultaneously break | Corrosion, Design Defection | No function can be implemented any more | Undetectable during PST | Dangerous | Dangerous |
| 15 | Gasket | Seal the body/bonnet | Break | Corrosion | ELP | Undetectable during PST | No effect | No effect |
| 16 | Bleeding & Drain | Bleed the body | Plugged | Pollution | Can't bleed | Undetectable during PST | No part | No part |
| 17 | Trunnion | Support the ball | Distortion | Material Defection, Over-pressure | DOP | Detectable during PST | Dangerous | Dangerous |

**Appendix B:** Calculation example of multiphase Markovian equations
(MTTR=24 hour; TI=2 year; PSTI=3 month)

$p := \dfrac{3 \cdot 720}{2} + 24$

$$1104$$

$T := (817 \cdot 0.\,95 - 6) \cdot 21900 + (877 \cdot 0.\,9 - 34) \cdot 13140 + (227 \cdot 0.\,85 - 67) \cdot 4380$

$$2.\,734258800\ 10^{7}$$

**1-3months**

$A$

$:= add\Bigg(\ [\,1,\ 0,\ 0,\ 0,$

$0\,]\cdot\begin{bmatrix} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.\,452\cdot 9}{T} - \dfrac{0.\,548\cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.\,452\cdot 9}{T} & \dfrac{0.\,548\cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{k},\ k$

$= 1\,..\,2160\Bigg)$

$$[\,2151.\,03985597549,\ 8.\,00130090465542,\ 0.\,340480889544950,$$
$$0.\,198570605645084,\ 0.\,419812936933905\,]$$

$a$

$:= [\,1,\ 0,\ 0,\ 0,$

$0\,]\cdot\begin{bmatrix} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.\,452\cdot 9}{T} - \dfrac{0.\,548\cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.\,452\cdot 9}{T} & \dfrac{0.\,548\cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{2160}$

$$[\,0.\,991762041096043,\ 0.\,00739500301976258,\ 0.\,000314680979550635,$$
$$0.\,000140292954066077,\ 0.\,000388001647901444\,]$$

$$B := add \left[ \begin{array}{l} [0.991762041096043, \ 0.00739500301976258, \end{array} \right.$$

$$0.000314680979550635, \ 0, \ 0.000388001647901444]$$

$$\left[ \begin{array}{ccccccccc} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.452 \cdot 9}{T} - \dfrac{0.548 \cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452 \cdot 9}{T} & \dfrac{0.548 \cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]^{k}, \ k = 1$$

$$..2160$$

$$[2133.31967804118, \ 23.9085930393119, \ 1.01738693779867,$$
$$0.196934789156246, \ 1.25443809467921]$$

$$b := [0.991762041096043, \ 0.00739500301976258, \ 0.000314680979550635,$$
$$0, \ 0.000388001647901444]$$

$$\left[ \begin{array}{ccccccccc} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.452 \cdot 9}{T} - \dfrac{0.548 \cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452 \cdot 9}{T} & \dfrac{0.548 \cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]^{2160}$$

$$[0.983591946158989, \ 0.0147290863085537, \ 0.000626769630123875,$$
$$0.000139137226475966, \ 0.000772806954172809]$$

**7-9 months**

$$C := add\left( \left[ 0.983591946158989, \ 0.0147290863085537, \right. \right.$$

$$0.000626769630123875, \ 0, \ 0.000772806954172809 \right]$$

$$\left[ \begin{array}{cccccccc} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.452 \cdot 9}{T} - \dfrac{0.548 \cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452 \cdot 9}{T} & \dfrac{0.548 \cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]^{k} , \ k = 1$$

$$\left. ..2160 \right)$$

$$\left[ 2115.74547820448, \ 39.6848415550897, \ 1.68871666184503, \right.$$
$$\left. 0.195312448456418, \ 2.08218764467480 \right]$$

$$c := \left[ 0.983591946158989, \ 0.0147290863085537, \ 0.000626769630123875, \right.$$
$$\left. 0, \ 0.000772806954172809 \right]$$

$$\left[ \begin{array}{cccccccc} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.452 \cdot 9}{T} - \dfrac{0.548 \cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452 \cdot 9}{T} & \dfrac{0.548 \cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]^{2160}$$

$$\left[ 0.975489156128268, \ 0.0220027517206136, \ 0.000936287307219301, \right.$$
$$\left. 0.000137991019722247, \ 0.00115444225014509 \right]$$

**10-12 months**

$$E := add\left(\left[0.975489156128268,\ 0.0220027517206136,\right.\right.$$

$$0.000936287307219301,\ 0,\ 0.00115444225014509]$$

$$\left.\begin{bmatrix} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.452 \cdot 9}{T} - \dfrac{0.548 \cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452 \cdot 9}{T} & \dfrac{0.548 \cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{k},\ k = 1\right.$$

$$\left.\left..\,.\,2160\right)\right.$$

$$[2098.31605390380,\ 55.3311259839361,\ 2.35451599921369,$$
$$0.193703472532602,\ 2.90311822789478]$$

$$c := [0.975489156128268,\ 0.0220027517206136,\ 0.000936287307219301,$$
$$0,\ 0.00115444225014509]$$

$$\begin{bmatrix} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.452 \cdot 9}{T} - \dfrac{0.548 \cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452 \cdot 9}{T} & \dfrac{0.548 \cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{2160}$$

$$[0.967453116548828,\ 0.0292164969759278,\ 0.00124325519041077,$$
$$0.000136854255372660,\ 0.00153293365023285]$$

**13-15 months**

$$F := add\left(\begin{array}{l}[0.967453116548828,\ 0.0292164969759278,\\[10pt] 0.00124325519041077,\ 0,\ 0.00153293365023285]\end{array}\right.$$

$$\begin{bmatrix} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.452\cdot9}{T} - \dfrac{0.548\cdot9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452\cdot9}{T} & \dfrac{0.548\cdot9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{k},\ k = 1$$

$$\left.\phantom{x}..2160\right)$$

$$[2081.03021248422,\ 70.8485169646579,\ 3.01483050900285,$$
$$0.192107751286326,\ 3.71728601870718]$$

$$f := [0.967453116548828,\ 0.0292164969759278,\ 0.00124325519041077,\ 0,$$
$$0.00153293365023285]$$

$$\begin{bmatrix} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.452\cdot9}{T} - \dfrac{0.548\cdot9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452\cdot9}{T} & \dfrac{0.548\cdot9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{2160}$$

$$[0.959483277533194,\ 0.0363708156942851,\ 0.00154769428479567,$$
$$0.000135726855641068,\ 0.00190830705372118]$$

**16-18 months**

$$G := \text{add}\left(\left[0.959483277533194,\ 0.0363708156942851,\right.\right.$$

$$0.00154769428479567,\ 0,\ 0.00190830705372118]$$

$$\left.\left[\begin{array}{cccccccc} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.452 \cdot 9}{T} - \dfrac{0.548 \cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452 \cdot 9}{T} & \dfrac{0.548 \cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]^{k},\ k = 1 \right.$$

$$\left.\ .. 2160 \right)$$

$$[2063.88677111589,\ 86.2380763161839,\ 3.66970537499665,$$
$$0.190525175526097,\ 4.52474672871794]$$

$$g := [0.959483277533194,\ 0.0363708156942851,\ 0.00154769428479567,\ 0,$$
$$0.00190830705372118]$$

$$\left[\begin{array}{cccccccc} 1 - \dfrac{94}{T} - \dfrac{4}{T} - \dfrac{0.452 \cdot 9}{T} - \dfrac{0.548 \cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452 \cdot 9}{T} & \dfrac{0.548 \cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1 - \dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]^{2160}$$

$$[0.951579093723842,\ 0.0434661974290548,\ 0.00184962542243227,$$
$$0.000134608743382134,\ 0.00228058814653794]$$

**19-21 months**

$$H := add\left(\begin{array}{l}[0.951579093723842,\ 0.0434661974290548,\end{array}\right.$$

$$0.00184962542243227,\ 0,\ 0.00228058814653794]$$

$$\left.\begin{bmatrix} 1-\dfrac{94}{T}-\dfrac{4}{T}-\dfrac{0.452 \cdot 9}{T}-\dfrac{0.548 \cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452 \cdot 9}{T} & \dfrac{0.548 \cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1-\dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{k},\ k=1\right.$$

$$\left. \phantom{x} \right)$$
$$..2160$$

$$[2046.88455671302,\ 101.500857110222,\ 4.31918540875717,$$
$$0.188955636959945,\ 5.32555561058307]$$

$$h := [0.951579093723842,\ 0.0434661974290548,\ 0.00184962542243227,\ 0,$$
$$0.00228058814653794]$$

$$\begin{bmatrix} 1-\dfrac{94}{T}-\dfrac{4}{T}-\dfrac{0.452 \cdot 9}{T}-\dfrac{0.548 \cdot 9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452 \cdot 9}{T} & \dfrac{0.548 \cdot 9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1-\dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{2160}$$

$$[0.943740024255880,\ 0.0505031277006855,\ 0.00214906926376519,$$
$$0.000133499842086038,\ 0.00264980240301135]$$

**22-24 months**

$$J := add\left(\begin{array}{l}[0.943740024255880,\ 0.0505031277006855,\end{array}\right.$$

$$0.00214906926376519,\ 0,\ 0.00264980240301135]$$

$$\left[\begin{array}{cccccccc} 1-\dfrac{94}{T}-\dfrac{4}{T}-\dfrac{0.452{\cdot}9}{T}-\dfrac{0.548{\cdot}9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452{\cdot}9}{T} & \dfrac{0.548{\cdot}9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1-\dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]^{k},\ k=1$$

$$..2160\Bigg)$$

$$[2030.02240585367,\ 116.637903743319,\ 4.96331505269061,$$
$$0.187399028187997,\ 6.11976746178946]$$

$$j := [0.943740024255880,\ 0.0505031277006855,\ 0.00214906926376519,\ 0,$$
$$0.00264980240301135]$$

$$\left[\begin{array}{cccccccc} 1-\dfrac{94}{T}-\dfrac{4}{T}-\dfrac{0.452{\cdot}9}{T}-\dfrac{0.548{\cdot}9}{T} & \dfrac{94}{T} & \dfrac{4}{T} & \dfrac{0.452{\cdot}9}{T} & \dfrac{0.548{\cdot}9}{T} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \dfrac{1}{p} & 0 & 0 & 1-\dfrac{1}{p} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]^{2160}$$

$$[0.935965532720041,\ 0.0574820880299285,\ 0.00244604629903917,$$
$$0.000132400075873249,\ 0.00301597508761318]$$

**Preliminary calculation**

$$\dfrac{\left(\dfrac{(A+B+C+E+F+G+H+J)}{2160}\right)}{8}$$

$$[0.967606771544661,\ 0.0290596768297093,\ 0.00123658199269963,$$
$$0.0000893234321614997,\ 0.00152470559745257]$$

**PFDavg calculation for SIF a&b**

$$0.0000893234321614997 + 0.00152470559745257$$

**0.007818795991**

## PFDavg calculation for SIF c

$$0.0000893234321614997 + 0.00152470559745257 + 0.00123658199269963$$

**0.008071594209**